

## MID-INFRARED PERIOD-METALLICITY-LUMINOSITY RELATIONS AND KINEMATICS OF RR LYRAE VARIABLES

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### ABSTRACT

We use ALLWISE data release W1- and W2-band epoch photometry collected by the Wide-Field Infrared Survey Explorer (WISE) to determine slopes of the period-luminosity relations for RR Lyrae stars in 15 globular clusters in the corresponding bands. We further combine these results with V- and K-band photometry of Galactic field RR Lyrae stars to determine the metallicity slopes of the  $\log P_F - [\text{Fe}/\text{H}] - M_K$ ,  $\log P_F - [\text{Fe}/\text{H}] - M_{W1}$ , and  $\log P_F - [\text{Fe}/\text{H}] - M_{W2}$  period-metallicity-luminosity relations. We infer the zero points of these relations and determine the kinematical parameters of thick-disk and halo RR Lyraes via statistical parallax, and estimate the RR Lyrae-based distances to 18 Local-Group galaxies including the center of the Milky Way.

*Key words:* stars: variables: RR Lyrae, techniques: photometric

## 1. INTRODUCTION

Luminosities of RR Lyrae variables in photometric band X depend on their fundamental periods and metallicity (Catelan et al. , 2004):

$$\langle M_X \rangle = \alpha_X \cdot \log P_F + \beta_X \cdot [\text{Fe}/\text{H}] + \gamma_X, \quad (1)$$

where  $\langle M_X \rangle$  is the intensity-mean absolute magnitude in photometric band X;  $P_F$  the fundamental-mode period, and  $[\text{Fe}/\text{H}]$  the metallicity. The fundamental period  $P_F$  is equal to the variability period  $P$  for RRab type variables (fundamental-mode pulsators) and  $\log P_F = \log P + 0.127$  or  $P_F = P/0.746$  for RRc type variables (first-overtone pulsators). Relation (1) is rather tight (with a scatter of  $\sim 0.05^m - 0.10^m$ ), making RR Lyraes highly efficient standard candles for estimating distances to stellar systems with old populations. Our aim is to determine the parameters of the period-metallicity-luminosity relation (1) in the mid-infrared for RR Lyrae based on photometric data for such variables in globular clusters.

## 2. THE DATA

Our source for mid-infrared photometry is data acquired from the WISE all-sky photometric survey (Wright et

al. , 2010). Here we use the ALLWISE data release to compute the intensity-mean WISE W1- and W2-band magnitudes of RR Lyrae variables in globular clusters and estimate the period slope  $\alpha$ , employ photometric and metallicity data for field RR Lyraes (homogenized intensity-mean V-band magnitudes and metallicities from Dambis et al. (2013) and W1- and W2-band intensity mean magnitudes from Gavrilchenko et al. (2014)) to derive the metallicity slope  $\beta$ , and use the results of our statistical-parallax study (Dambis et al. , 2013) and HST FSG trigonometric parallaxes to estimate the zero point  $\gamma$  of the WISE W1- and W2-band relations (1).

## 3. PERIOD SLOPES $\alpha$

In view of Equation (1), the apparent X-band magnitude  $\langle X \rangle$  of an RR Lyrae type star in a particular cluster is given by the following formula:

$$\langle X \rangle = \alpha_X \log P_F + \beta_X [\text{Fe}/\text{H}] + \gamma_X + (m - M)_0 + A_X, \quad (2)$$

where  $(m - M)_0$  and  $A_X$  are the true distance modulus and the total X-band extinction, respectively. We can rewrite the above equation as

$$\langle X \rangle - \alpha_X (\log P_F + 0.25) = C_X$$

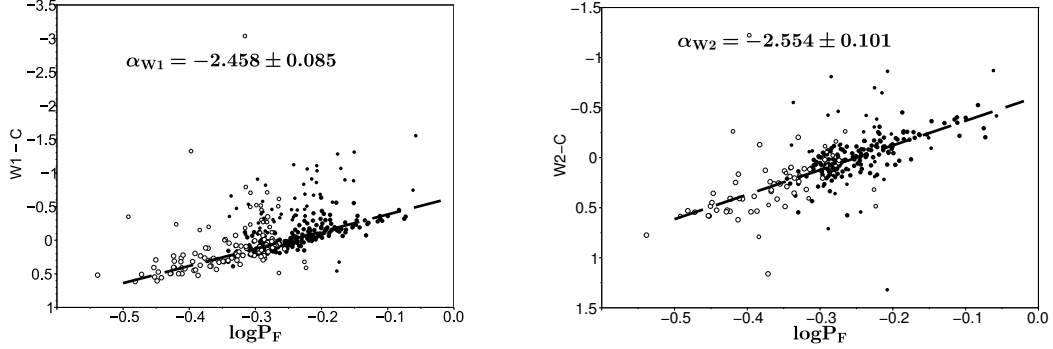


Figure 1. Combined period-W1-band magnitude and period-W2-band magnitude relations for RR Lyraes in 15 and 8 globular clusters, respectively.

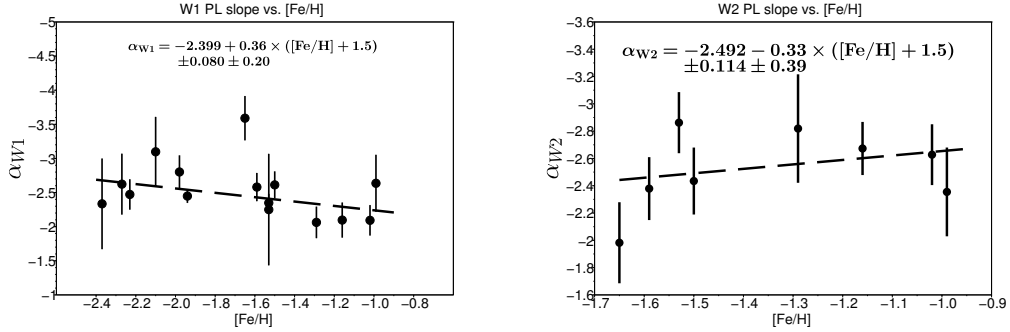


Figure 2. The period slopes  $\alpha_{W1}$  and  $\alpha_{W2}$  as functions of cluster metallicity.

or

$$\langle X \rangle = \alpha_X (\log P_F + 0.25) + C_X \quad (3)$$

where  $C_X = \beta_X [Fe/H] + \gamma_X + (m - M)_0 + A_X - 0.25\alpha_X$  can be considered to be a constant for all RR Lyrae variables of a particular cluster.

Figure 1 shows the combined W1- and W2-band relations (3) for RR Lyrae variables in 15 and 8 globular clusters, respectively. The average slopes are equal to  $\alpha_{W1} = -2.458 \pm 0.085$  and  $\alpha_{W2} = -2.554 \pm 0.101$ , respectively. We show individual cluster slope estimates plotted against  $[Fe/H]$  in Fig. 2.

#### 4. METALLICITY SLOPES $\beta$

We now follow the procedure employed by Dambis et al. (2013) to estimate the metallicity slopes  $\beta_{W1}$  and  $\beta_{W2}$  of the W1- and W2-band PML relations for RR Lyraes. First, we note that the V-band absolute magnitude of RR Lyraes is a function of metallicity and, at fixed  $[Fe/H]$ , is independent of period (see, e.g., Catealan et al. (2004)). We adopt the following form of the  $M_V = [Fe/H]$  dependence:

$$\langle M_V \rangle = \gamma_V + 0.232 \cdot [Fe/H], \quad (4)$$

where the slope is the simple average of direct estimates by Gratton et al. (2004) and Federici et al. (2012) based on the observed magnitudes of RR Lyrae in the LMC and horizontal-branch stars in M31 globular clus-

ters, respectively. Hence the apparent V-band magnitude  $\langle V \rangle$  of an RR Lyrae type star is equal to

$$\langle V \rangle = \gamma_V + 0.232 \cdot [Fe/H] + (m - M)_0 + A_V, \quad (5)$$

where  $(m - M)_0$  and  $A_V$  are the true distance modulus and the V-band interstellar extinction toward the star, respectively. We now subtract Equation (2) from Equation (5) to obtain:

$$\begin{aligned} \langle V \rangle - \langle X \rangle &= (\gamma_V - \gamma_X) - \alpha_X \log P_F \\ &+ (0.232 - \beta_X) \cdot [Fe/H] + A_V - A_X, \end{aligned} \quad (6)$$

or

$$\begin{aligned} \langle V \rangle - \langle X \rangle + \alpha_X \log P_F - A_V + A_X &= \\ (\gamma_V - \gamma_X) + (0.232 - \beta_X) \cdot [Fe/H]. \end{aligned} \quad (7)$$

To determine the metallicity slopes  $\beta_{W1}$  and  $\beta_{W2}$  for W1- and W2-band PML relations, we use 265 stars at Galactic latitudes  $|b| \geq +25^\circ$  drawn from the list of Dambis et al. (2013) with bona fide photoelectric or CCD intensity-mean V-band magnitudes and homogeneous  $[Fe/H]$  values. The intensity-mean W1- and W2-band magnitudes of these stars based on ALLWISE individual-epoch photometry are available from Gavrilchenko et al. (2014) and the V-, W1-, and W2-band extinction ( $A_V$ ,  $A_{W1}$ , and  $A_{W2}$ ) toward them can be computed from the 3D extinction map of Drimmel et

Table 1  
KINEMATICAL PARAMETERS AND W1-BAND ABSOLUTE-MAGNITUDE CORRECTION OF GALACTIC FIELD RR LYRAE  
VARIABLES BASED ON THE BIMODAL SOLUTION  
(336 STARS WITH GALACTOCENTRIC DISTANCES IN THE INTERVAL FROM 6.4 TO 9.6 KPC).

Population	Fraction	$U_0$	$V_0$	$W_0$	$\sigma V_R$ km/s	$\sigma V_\theta$	$\sigma W$	$\gamma_{W1}$
Halo	0.782 $\pm 0.029$	-7.8 $\pm 9.1$	-211.8 $\pm 9.5$	-9.2 $\pm 9.1$	151.1 $\pm 8.3$	100.9 $\pm 5.7$	97.2 $\pm 5.4$	$-0.825 \pm 0.088$
Disk	0.218 $\pm 0.029$	-10.8 $\pm 6.8$	-37.1 $\pm 6.0$	-16.9 $\pm 4.0$	45.6 $\pm 6.7$	36.1 $\pm 5.1$	26.9 $\pm 4.2$	

al. (2003) and the reddening law of Yuan et al. (2013). Hence the right-hand side quantities in Equation (7) for  $X = W1$  and  $X = W2$  are known for these stars and so are the coefficients  $[Fe/H]$  in the right-hand side part of the corresponding equations. We solve Equations (7) for  $X = W1$  and  $X = W2$  for parameters  $(\gamma_V - \gamma_{W1})$  and  $(0.232 - \beta_{W1})$  to obtain:

$$\begin{aligned} & (\langle V \rangle - \langle W1 \rangle)_0 = \\ & = 1.936 + 0.130 \cdot [Fe/H] + 2.458 \cdot \log P_F \end{aligned} \quad (8)$$

and

$$\begin{aligned} & (\langle V \rangle - \langle W2 \rangle)_0 = \\ & 1.956 + 0.139 \cdot [Fe/H] + 2.554 \cdot \log P_F, \end{aligned} \quad (9)$$

implying  $\beta_{W1} = 0.102 \pm 0.023$  and  $\beta_{W2} = 0.099 \pm 0.023$ , respectively. Hence

$$\langle M_{W1} \rangle = \gamma_{W1} + 0.102[Fe/H] - 2.458 \cdot \log P_F \quad (10)$$

and

$$\langle M_{W2} \rangle = \gamma_{W2} + 0.099[Fe/H] - 2.554 \cdot \log P_F. \quad (11)$$

## 5. ZERO POINTS $\gamma$

### 5.1. Calibration via HST FGS trigonometric parallaxes

We first calibrate the zero points  $\gamma_{W1}$  and  $\gamma_{W2}$  using the HST FGS trigonometric parallaxes of four RR Lyraes from Benedict et al. (2007) to obtain

$$\langle M_{W1} \rangle = -1.160 - 2.458 \cdot \log P_F + 0.102 \cdot [Fe/H], \quad (12)$$

and

$$\langle M_{W2} \rangle = -1.176 - 2.554 \cdot \log P_F + 0.099 \cdot [Fe/H]. \quad (13)$$

The corresponding relations for the V- and K-band absolute magnitudes calibrated via HST FGS trigonometric parallaxes are:

$$\langle M_K \rangle = -1.087 - 2.38 \cdot \log P_F + 0.088 \cdot [Fe/H], \quad (14)$$

and

$$\langle M_V \rangle = +0.776 + 0.232 \cdot [Fe/H]. \quad (15)$$

### 5.2. Calibration via the Method of Statistical Parallax

We now use the maximum-likelihood version of the method of statistical parallax suggested by Murray (1983) (pp. 297-302) adjusted for the case of a bimodal velocity distribution (Dambis, 2009) to determine the distance-scale correction factor and kinematical parameters of a sample of 336 RR Lyrae type variables in a Galactocentric distance interval from 6.4 to 9.6 kpc. We computed the initial distances of RR Lyraes using the PML relation (12), W1 intensity-mean magnitudes adopted from Gavrilchenko et al. (2014), and published  $[Fe/H]$  values homogenized by Dambis et al. (2013). The input kinematical observational data included UCAC4 proper motions and radial velocities from Table 2 of Dambis et al. (2013). The results are listed in Table 1. Here  $U_0$ ,  $V_0$ , and  $W_0$  are the components of the local mean solar velocity toward the Galactic center, in the direction of Galactic rotation, and toward the North Galactic Pole, respectively, relative to the RR Lyrae sample;  $\sigma V_R$ ,  $\sigma V_\theta$ , and  $\sigma W$  are the diagonal components of the velocity dispersion tensor in the directions of projected Galactocentric radius, Galactic rotation, and toward the North Galactic Pole, respectively, and  $\gamma_{W1}$  is the adjusted zero point of the W1-band PML relation. Hence

$$\langle M_{W1} \rangle = -0.825 - 2.458 \cdot \log P_F + 0.102 \cdot [Fe/H], \quad (16)$$

and, in view of Equations (8) and (9):

$$\langle M_V \rangle = +1.111 + 0.232 \cdot [Fe/H]. \quad (17)$$

and

$$\langle M_{W2} \rangle = -0.841 - 2.554 \cdot \log P_F + 0.099 \cdot [Fe/H]. \quad (18)$$

The corresponding relation for the K-band absolute magnitudes calibrated via statistical parallax is:

$$\langle M_K \rangle = -0.752 - 2.38 \cdot \log P_F + 0.088 \cdot [Fe/H]. \quad (19)$$

As is evident from a comparison of eqs. (12)–(15) and (16)–(19), the RR Lyrae distance scale based on Hubble Space Telescope Fine Guidance Sensor parallaxes is  $\sim 0.3^m$  longer than the distance scale with zero point inferred via statistical parallax. This discrepancy remains a long-standing issue and one apparently has to wait for GAIA results for it to be finally resolved. The distance estimates reported below are all given in two versions – for each of the two distance-scale zero points.

Table 2  
DISTANCES TO LOCAL GROUP GALAXIES BASED ON  
PUBLISHED V- AND K-BAND PHOTOMETRY OF THEIR RR  
LYRAE TYPE VARIABLES.

Galaxy	Distance (stat. par.)	Distance (HST FGS)	Data
Milky Way (Galactic center)	$7.7 \pm 0.4$	$9.0 \pm 0.4$	1,2,3
LMC	$46 \pm 2$	$54 \pm 2$	4,5,6,7
SMC	$57 \pm 2$	$66 \pm 2$	8,9
M31	$703 \pm 30$	$823 \pm 35$	10,11
M32	$711 \pm 40$	$830 \pm 37$	12
NGC147	$631 \pm 48$	$736 \pm 56$	13
And I	$703 \pm 36$	$820 \pm 42$	14
And II	$611 \pm 32$	$713 \pm 37$	15
And III	$685 \pm 35$	$799 \pm 41$	14
And V	$762 \pm 32$	$889 \pm 37$	16
And VI	$748 \pm 32$	$873 \pm 37$	17
And XI	$682 \pm 39$	$896 \pm 46$	18
And XIII	$780 \pm 33$	$910 \pm 39$	18
M33	$745 \pm 31$	$869 \pm 36$	19
IC1613	$689 \pm 29$	$804 \pm 34$	20
Sex dSph	$89 \pm 9$	$104 \pm 10$	21
Cetus	$748 \pm 32$	$873 \pm 37$	22
Tucana	$855 \pm 36$	$1000 \pm 42$	22

Data sources: 1. Carney et al. (1995);  
2. Groenewegen et al. (2008);  
3. Collinge, Sumi & Fabrycky (2006);  
4. Borissova et al. (2004); 5. Szweczyk et al. (2008);  
6. Dall’Ora et al. (2004); 7. Gratton et al. (2004);  
8. Kapakos & Hatzidimitriou (2012);  
9. Szweczyk et al. (2009); 10. Clementini et al. (2009);  
11. Federici et al. (2012); 12. Sarajedini et al. (2012);  
13. Yang & Sarajedini (2010); 14. Pritzl et al. (2005);  
15. Pritzl et al. (2004); 16. Mancone & Sarajedini (2008);  
17. Pritzl et al. (2002); 18. Yang & Sarajedini (2012);  
19. Yang et al. (2010); 20. Bernard et al. (2010);  
21. Lee et al. (2003); 22. Bernard et al. (2009).

## 6. DISTANCES TO LOCAL GROUP GALAXIES

We use PML relations (17) and (19) – and the corresponding relations (15) and (14) – to estimate the distances to a number of Local group galaxies based on photometric data for RR Lyrae type variables and HB stars. The results are summarized in Table 2.

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