

## A NEW NON-PARAMETRIC APPROACH TO DETERMINE PROPER MOTIONS OF STAR CLUSTERS

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### ABSTRACT

The bulk motion of star clusters can be determined after careful membership analysis using parametric or non-parametric approaches. This study aims to implement non-parametric membership analysis based on Binned Kernel Density Estimators which takes into account measurements errors (simply called BKDE-e) to determine the average proper motion of each cluster. This method is applied to 178 selected star clusters with angular diameters less than 20 arcminutes. Proper motion data from UCAC4 are used for membership determination. Non-parametric analysis using BKDE-e successfully determined the average proper motion of 129 clusters, with good accuracy. Compared to COCD and NCOVOCC, there are 79 clusters with less than  $3\sigma$  difference. Moreover, we are able to analyse the distribution of the member stars in vector point diagrams which is not always a normal distribution.

*Key words:* proper motion: open clusters and associations

### 1. INTRODUCTION

In the study of stellar cluster population in the Galactic disk, membership analysis or decontamination process is an important process prior to physical, astrometric and photometric analysis. This process can be conducted based on proper motion data through parametric (Zhao and He 1990) or non-parametric (Cabrera-Caño and Alfaro 1990) methods. The emergence of non-parametric approaches for membership analysis is mainly due to the lack of parametric functions to match the actual distribution of proper motion data over vector point diagram (VPD). The distribution does not always conform to a normal distribution as assumed in the parametric approach. Thus, non-parametric density estimation (e.g. Kernel Density Estimation, KDE) serves as an alternative method to analyse the kinematic distribution of cluster members and finally decontaminate them of field stars.

However, kernel-based non-parametric methods usually require more computational effort ( $O(n^2)$ ), especially in the age of all-sky surveys. Binning scheme as discussed by Wand (1994) may provide a way out. The so-called Binned Kernel Density Estimation (BKDE-e Priyatikanto and Arifyanto) is used in this study to decontaminate the star clusters of non-member stars and also to determine the bulk motion of the cluster in the celestial plane. This method takes measurement errors into consideration.

### 2. BKDE-e IN BRIEF

In BKDE-e membership analysis (Priyatikanto and Arifyanto), the proper motion data are binned using linear binning. The number of bins is determined appropriately to minimize error in the density estimation (Wand 1994). After binning, the kernel density estimation is conducted for each knots to model the kinematic distribution of stars within the sampling radius ( $f_{c+f}$ ) and the field stars in the annulus ( $f_f$ ). Then, the membership probability can be calculated using:

$$P = \frac{f_{c+f} - f_f}{f_{c+f}}.$$

In the BKDE-e ( $1D$  case), the density estimate for each knot (binning representative,  $x_{g,i}$ ) is calculated using:

$$f(x_g) = \frac{1}{nH} \sum_{i=1}^{n_{g,x}} K\left(\frac{x_g - x_{g,i}}{h_x}\right) c_{g,i}, \quad (1)$$

where  $H$  is the kernel width,  $K(x)$  is the kernel function (a Gaussian is used here), while  $c_{g,i}$  is the count for each knot that represents the total weights from neighbouring data points. A linear binning scheme is used to obtain the count for each knot (see binning). To accommodate measurement errors, each data point give a partial weight to the related knots according to the total area enclosed by two knots:

$$w_{g,b} = \int_{x_b}^{x_c} g(x') dx' - \frac{1}{H} \int_{x_b}^{x_c} (x' - x_b) g(x') dx', \quad (2)$$

$$w_{g,c} = \frac{1}{H} \int_{x_b}^{x_c} (x' - x_b) g(x') dx'. \quad (3)$$

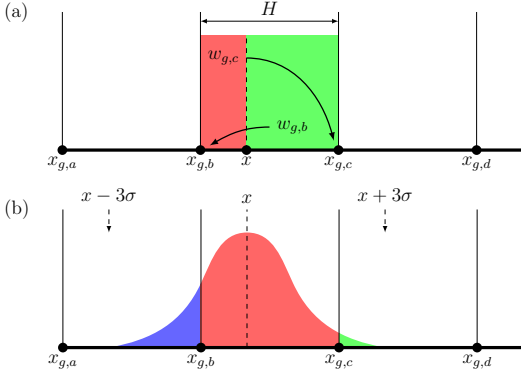


Figure 1. Linear binning scheme of 1D data point  $x \pm \sigma$  contained within  $H$ -sized bin. Instead of giving weight to two neighbouring knots ( $x_{g,b}$  and  $x_{g,c}$ ) (a), a data point with an error may give its weight to more than two knots enclosed by its  $3\sigma$  wings (b).

The bulk motion or average proper motion of the cluster is determined according to the mode of the kinematic distribution on the VPD. Stars with  $f_c > 90\% f_{c,max}$  are averaged to determine the average proper motion. This approach is more robust compared to the average proper motion of member stars because of the asymmetrical distribution.

### 3. KINEMATIC DATA

The Fourth US Naval Observatory CCD Astrograph Catalog (UCAC4 Zacharias et al., 2013), which includes  $\sim 105$  millions stars with proper motion data and typical errors less than 10 mas/yr is the source of the kinematic data in this study. We selected 178 open clusters with radius  $R < 1^\circ$  and  $1\sigma$  members  $N_1 > 10$  as catalogued by Kharchenko et al. (2005). Stars with photometric errors of  $e_J < 0.1$  and kinematic errors of  $\mu < 7$  mas/yr are used in membership analysis. The distances to the selected clusters range from 0.3 to 6.0 kpc, while the age range is  $\sim 4$  Myr to  $\sim 2$  Gyr.

For the selected stars of each cluster, we categorize them into *in-field* and *out-field* stars. In-field stars ( $r < R_{tide}$ ) consist of both member and field stars, while out-field stars ( $1.2R_{tide} < r < 1.6R_{tide}$ ) contain only field stars.

### 4. RESULTS AND DISCUSSION

Kinematic analyses were conducted on 178 selected clusters. Of these, we successfully obtained proper motion of 129 (72%) with a median uncertainty of 1.5 mas/yr. One example of the successful cases is NGC 2682 as displayed in *ngc2682*. The other 49 clusters (28%) have low concentrations or are embedded within the cloud. These conditions increase the difficulty in the decontamination process by the non-parametric approach. In most of the unsuccessful cases, density estimates of the in-field stars are indistinguishable compared to the out-field stars.

The obtained results were compared to the published proper motion catalogs: Catalog of Open Cluster Data (COCD Kharchenko et al., 2005) and New Cata-

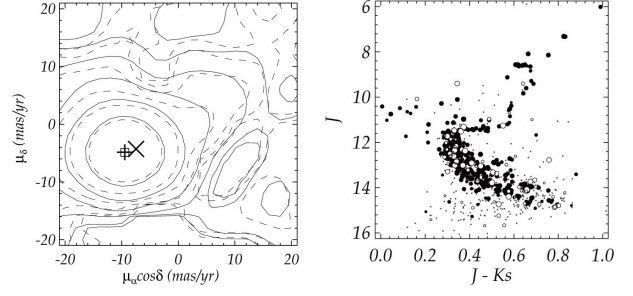


Figure 2. *Left*: Contour of density estimate (on VPD) of stars around NGC 2682 obtained using BKDE-e (solid) and ordinary KDE (dashed), plus marks the average proper motion obtained using BKDE-e. *Right*: the location of cluster members (filled circles) and non-members (open circles) over the CMD.

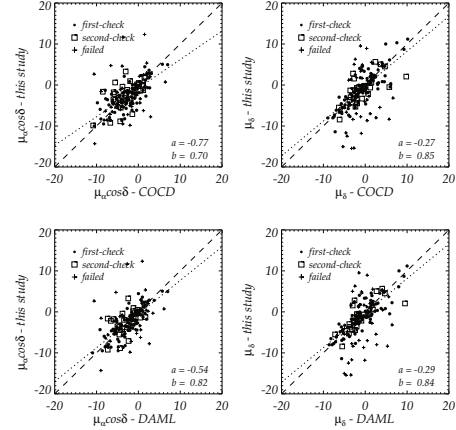


Figure 3. Plots of cluster proper motion obtained in this study compared to COCD (top) and NCOVOCC (bottom). Dotted lines mark the linear fit of the data showing a small deviation, fitting coefficients ( $a, b$ ) are displayed as well.

log of Optically Visible Open Clusters and Candidates (NCOVOCC Dias et al., 2002) (see compare). For quantitative comparison, we defined:

$$\Delta_\mu^2 = \frac{(\mu_x - \mu_{x,cat})^2}{\sigma_x^2 + \sigma_{x,c}^2} + \frac{(\mu_y - \mu_{y,cat})^2}{\sigma_y^2 + \sigma_{y,c}^2}, \quad (4)$$

where  $\mu_x = \mu_\alpha \cos \delta$  and  $\mu_y = \mu_\delta$ . Of the 129 successful cases, 61% of our results agree with the catalogs ( $\Delta_\mu < 3$ ).

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