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LOCATIONS OF OUT-OF-PLANE EQUILIBRIUM POINTS IN THE ELLIPTIC RESTRICTED THREE-BODY PROBLEM UNDER RADIATION AND OBLATENESS EFFECTS

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ABSTRACT

This study deals with the generalization of the Elliptic Restricted Three-Body Problem (ER3BP) by considering the effects of radiation and oblate spheroid primaries. This may illustrate a gas giant exoplanet orbiting its host star with eccentric orbit. In the three dimensional case, this generalization may possess two additional equilibrium points ($L_{6,7}$, out-of-plane). We determine the existence of $L_{6,7}$ in ER3BP under the effects of radiation (bigger primary) and oblateness (small primary). We analytically derive the locations of $L_{6,7}$ and assume initial approximations of $(\mu - 1, \pm\sqrt{3A_2})$, where μ and A_2 are the mass parameter and oblateness factor, respectively. The fixed locations are then determined. Our results show that the locations of $L_{6,7}$ are periodic and affected by A_2 and the radiation factor (q_1).

Key words: elliptic restricted three-body problem: out-of-plane: oblate spheroid: radiation

1. INTRODUCTION

The Classical Restricted Three-Body Problem (R3BP) has been generalized to include additional effects. The generalization of R3BP yields equilibrium points located out of the orbital plane (out-of-plane, $L_{6,7}$) (Douskos & Markellos, 2006). $L_{6,7}$ also have been found in the generalized Elliptic Restricted Three-Body Problem (ER3BP) (Singh & Umar, 2012). We study the locations of $L_{6,7}$ in the generalized ER3BP when considering effects of radiation only in the bigger primary (radiation factor: $q_1 \neq 1$, $q_2 = 1$) and an oblate spheroid only for the smaller primary (oblateness factor: $A_1 = 0, A_2 \neq 0$). This study is applicable for the determination of $L_{6,7}$ locations in exoplanetary systems which have a gas giant planet.

2. EQUATIONS OF MOTION

Let m_1 and m_2 be the masses of the primaries ($m_1 > m_2$). In the rotational frame, the coordinates of the third infinitesimal object are $(\xi,\ \eta,\ \zeta)$, and of the primaries are $(\xi_1,\ \eta_1,\ \zeta_1)$, and $(\xi_2,\ \eta_2,\ \zeta_2)$. The unit of time is chosen to make the gravitational constant $G = 1$. We use q_i and A_i to represent the radiation and oblateness coefficients of the bigger ($i = 1$) and smaller ($i = 2$) primaries, respectively, such that

$$A_i = \frac{{AE_i}^2 - {AP_i}^2}{5{R_i}^2}, \qquad q_i = 1 - \frac{F_{p_i}}{F_{g_i}},$$

with $0 < A_i \ll 1$ and $0 < 1 - q_i \ll 1$. AE_i and AP_i are the dimensional equatorial and polar radii, R_i is the effective radius, and F_{p_i} and F_{g_i} are the radiation pressure and gravitational attraction forces. In this paper we consider the case where only the bigger primary has a source of radiation ($q_1 \neq 1,\ q_2 = 1$) and the smaller primary is an oblate spheroid body ($A_1 = 0,\ A_2 \neq 0$). The equations of motion of the third object in a barycentric and dimensionless coordinate system ($\bar{\xi} = \frac{\xi}{r},\ \bar{\eta} = \frac{\eta}{r},\ \bar{\zeta} = \frac{\zeta}{r}$) due to the combined effect of oblateness and radiation are

$$\ddot{\bar{\xi}} - 2\dot{\bar{\eta}} = V_{\bar{\xi}}, \qquad \ddot{\bar{\eta}} + 2\dot{\bar{\xi}} = V_{\bar{\eta}}, \qquad \ddot{\bar{\zeta}} = V_{\bar{\zeta}}, \qquad (1)$$

where

$$V = (1 + e\cos f)^{-1}\left[\frac{1}{2}\left(\bar{\xi}^2 + \bar{\eta}^2 - \bar{\zeta}^2 e\cos f\right) + \frac{1}{n^2}\left[\frac{q_1(1-\mu)}{r_1} + \frac{\mu}{r_2}\left[1 + \frac{A_2}{2{r_2}^2}\left(1 - \frac{3\bar{\zeta}^2}{{r_2}^2}\right)\right]\right]\right],$$

and

$$\mu = m_2/(m_1 + m_2) \leq \frac{1}{2},$$

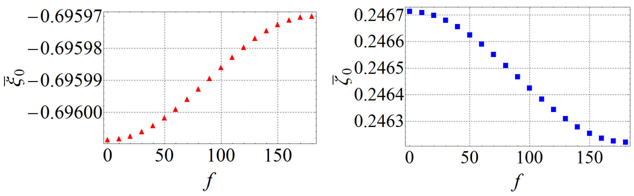

Figure 1. Relation between f and the position of L_6 ($\bar{\xi}_0$ (left), $\bar{\zeta}_0$ (right)). $\mu = 0.3$, $A_2 = 0.02$, $q_1 = 0.99$, and $e = 0.1$

$$
\begin{aligned}
n &= \sqrt{1 + \frac{3A_2}{2}}, \\
r_1 &= \sqrt{(\bar{\xi} - \mu)^2 + \bar{\eta}^2 + \bar{\zeta}^2}, \\
r_2 &= \sqrt{(\bar{\xi} - \mu + 1)^2 + \bar{\eta}^2 + \bar{\zeta}^2},
\end{aligned}
$$

where f is true anomaly, e is the eccentricity, and r is the distance between primaries. $\dot{\bar{\xi}}$, $\dot{\bar{\eta}}$, $\dot{\bar{\zeta}}$, $\ddot{\bar{\xi}}$, $\ddot{\bar{\eta}}$, and $\ddot{\bar{\zeta}}$ denote the partial derivatives with respect to f.

3. LOCATION OF OUT-OF-PLANE POINTS

The locations of $L_{6,7}$ can be obtained by imposing conditions as follow:

$$
\begin{aligned}
\dot{\bar{\xi}} = \dot{\bar{\eta}} = \dot{\bar{\zeta}} &= 0, \\
\bar{\xi}, \bar{\zeta} &\neq 0, \\
\bar{\eta} &= 0. \qquad (2)
\end{aligned}
$$

By substituting Equation 2 into Equation 1 and considering ${r_1}^2 = 1 + 3A_2$ and ${r_2}^2 = 3A_2$ we obtain

$$
\begin{aligned}
\bar{\xi}_0 &= \frac{(\mu - 1)\left[n^2 e\cos f + \frac{\mu}{3\sqrt{3}A_2^{3/2}} + \frac{q_1}{(3A_2+1)^{3/2}}\right]}{n^2(e\cos f + 1) + \frac{\mu}{3\sqrt{3}A_2^{3/2}}}, \\
\bar{\zeta}_0 &= \pm\sqrt{3A_2}\left[\frac{3\sqrt{3}(3A_2+2)A_2^{3/2}e\cos f}{5\mu}\right. \\
&\quad \left. - \frac{6\sqrt{3}A_2^{3/2}(\mu - 1)q_1}{5\mu(3A_2+1)^{3/2}} + 1\right]^{1/2}, \qquad (3)
\end{aligned}
$$

with ($\bar{\xi}_0$, $+\bar{\zeta}_0$) the position of L_6 and ($\bar{\xi}_0$, $-\bar{\zeta}_0$) the position of L_7. We determine that the existence of f in Equation 3 makes the locations of $L_{6,7}$ in ER3BP periodic. Equation 3 also shows that the locations of $L_{6,7}$ are affected by q_1 and A_2. An increasing value of A_2 produces locations of $L_{6,7}$ that are farther from the orbital plane, while reducing q_1 produces locations closer to the orbital plane. The relation between f and the position of L_6 are shown in Figures 1 and 2. Following Douskos & Markellos (2006) we also calculate the position of L_6 using numerical methods. We use the software package *Mathematica* and apply $\bar{\xi}_0 = \mu - 1$ and $\bar{\zeta}_0 = \pm\sqrt{3A_2}$ as the initial approximations. The comparison between analytical and numerical results is shown in Figure 3. It shows that both the numerical and analytical methods have almost the same result.

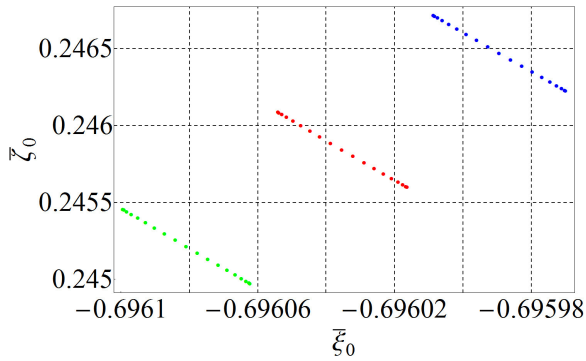

Figure 2. Position of L_6 ($\bar{\xi}_0$, $\bar{\zeta}_0$) as a function of f, with $\mu = 0.3$, $q_1 = 0.99$, $e = 0.1$, and A_2: 0.0198 (●), 0.0199 (●), and 0.0200 (●)

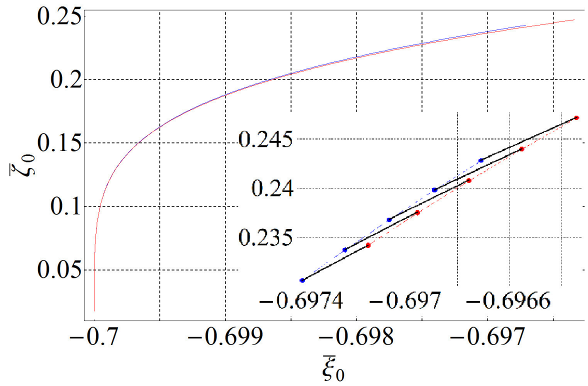

Figure 3. The position of L_6 ($\bar{\xi}_0$, $\bar{\zeta}_0$) as a function of A_2[0.0001, 0.02] with $\mu = 0.3$, $q_1 = 1.0$, $e = 0.4$, and $f = 45^o$. Numerical: -,●; Analytical: -,●

4. CONCLUSIONS

We determine that the locations of $L_{6,7}$ in ER3BP are periodic and affected by A_2 and q_1. Our study suggests that the increasing the value of A_2 produces the locations of $L_{6,7}$ that are farther from orbital plane, while reducing q_1 produces locations are closer to the orbital plane.

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