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LINEAR STABILITY OF TRIANGULAR EQUILIBRIUM POINTS IN THE PHOTOGRAVITATIONAL RESTRICTED THREE BODY PROBLEM WITH TRIAXIAL RIGID BODIES, WITH THE BIGGER ONE AN OBLATE SPHEROID AND SOURCE OF RADIATION

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ABSTRACT

In this paper we have examined the linear stability of triangular equilibrium points in the photogravitational restricted three body problem when both primaries are triaxial rigid bodies, the bigger one an oblate spheroid and source of radiation. The orbits about the Lagrangian equilibrium points are important for scientific investigation. A number of space missions have been completed and some are being proposed by various space agencies. We analyze the periodic motion in the neighbourhood of the Lagrangian equilibrium points as a function of the value of the mass parameter. Periodic orbits of an infinitesimal mass in the vicinity of the equilibrium points are studied analytically and numerically. The linear stability of triangular equilibrium points in the photogravitational restricted three body problem with Poynting-Robertson drag when both primaries are oblate spheroids has been examined by A. Kumar (2007). We have found the equations of motion and triangular equilibrium points for our problem. With the help of the characteristic equation we have discussed stability conditions. Finally, triangular equilibrium points are stable in the linear sense. It is further seen that the triangular points have long or short periodic elliptical orbits in the same range of μ.

Key words: Stability: Equilibrium Points: Photogravitational RTBP: Triaxial Rigid Bodies/ Oblate Spheroid

1. INTRODUCTION

The restricted problem three body problem possesses five equilibrium points, three collinear and two triangular. In the linear sense, the collinear points L_1, L_2, L_3 are unstable for any value of the mass ratio and the triangular points L_4, L_5 are stable if the mass ratio μ of the finite bodies is less than $\mu_0 = 0.03852\ldots\ldots$ Szebehely (1967). It is well known that in a sun-planet system, the motion of a material particle is subject to the solar radiation pressure. There are numerous examples in stellar systems too, in which either one or both the primaries are the source of radiation. Therefore, it is quite reasonable to modify the model of the restricted problem by taking the radiation pressure of the primaries into account. This modified model is known as the photogravitational restricted three body problem. In recent times many perturbing forces such as the oblateness and radiation forces of the primaries, coriolis and centrifugal forces, the variation of the masses of primaries and of the infinitesimal mass etc, have been included in the study of the restricted three body problem. In the case of the restricted three body problem where both the primaries are oblate spheroids whose equatorial planes coincide with the plane of motion, the location of libration points and their stability in the Liapunov sense have been studied by Vidyakin (1974). For the case, where the bigger primary is an oblate spheroid whose equatorial plane coincides with the plane of motion. P.V.Subba and R.K.Sharma (1975) have studied the stability of the libration points. A similar problem has been studied by El-Shaboury (1991). Khanna and Bhatnager (1998, 1999) have studied the problem when the smaller primary is a triaxial rigid body. B Ishwar (1997) studied the problem of non-linear stability in the generalized restricted three-body problem. Avdhesh Kumar and B. Ishwer (2008) have studied the linear stability of equilibrium points in the photogravitational restricted three body problem with Poynting-Robertson drag when both primaries are oblate spheroids.

In this paper, we have studied the linear stability of triangular equilibrium points in the photogravitational restricted three body problem when both primaries are triaxial rigid bodies, the bigger one an oblate spheroid and source of radiation with one of the axes as the axis of symmetry and the equatorial plane coinciding with the plane of motion. Further, we assume that the primaries are moving without rotation in circular orbits around their centre of mass.

2. EQUATIONS OF MOTION

Let m_1 and m_2 be the masses of the bigger and smaller primaries. The distance between the primaries does not change and is taken as unity, the sum of the masses of the primaries is also taken as unity. The unit of time is so chosen as to make the gravitational constant unity. Using dimensionless variables, the equations of motion of infinitesimal mass m_3 in a synodic co–ordinate system (x, y) are

$$\ddot{x} - 2n\dot{y} = \Omega_x \qquad (1)$$

$$\ddot{y} - 2n\dot{x} = \Omega_y \qquad (2)$$

where

$$\Omega = \sum [\frac{1}{2}n^2\mu_i r_i^2 + \frac{\omega_i}{r_i} + \frac{\omega_i}{2r_i^3}(2\sigma_{1i} - \sigma_{2i}) \qquad (3)$$

$$-\tfrac{3\omega_i}{2r_i^5}(\sigma_{1i} - \sigma_{2i})\,y^2] + \tfrac{\mu_1}{2r_i^3}A_1$$

$\sigma_{1i} = B_{1i} - B_{3i}, \quad \sigma_{2i} = B_{2i} - B_{3i} (i = 1, 2)$
$\omega_1 = (1-p)(1-\mu), \quad \omega_2 = \mu,$
$B_{1i} = \frac{a_i^2}{5R^2}, B_{2i} = \frac{b_i^2}{5R^2}, B_{3i} = \frac{c_i^2}{5R^2} (i = 1, 2)$

$a_i, b_i, c_i (i = 1, 2)$ is the length of its semi-axis, R is the distance between the primaries and the mean motion is given as in the equation.

3. STABILITY OF TRIANGULAR EQUILIBRIUM POINTS

Let the co-ordinate of the triangular point $L_{4,5}$ be denoted by (x_0, y_0). ξ, η denote a small displacement of the third body from L_4.

$\mu_{crit} = 0.038520895 - -+0.81126474\sigma_{11} - 1.09626653\sigma_{21} - 0.02206859\sigma_{12} - 0.04071097\sigma_{22} - 0.1330773375A_1 - 0.0089174706p$

$tan2\alpha = \frac{N}{D}$

$N = -\frac{3\sqrt{3}}{2}\left\{\frac{1}{24(1-\mu)}(36 - 65\mu + 37\mu^2)\sigma_{22} - \frac{1}{4}(7 - 10\mu)A_1\right\}$

$$n^2 = 1 + \sum_{i=1}^{2}\left[\frac{3}{2}(2\sigma_{1i} - \sigma_{2i})\right] + \frac{3}{2}A_1 \qquad (4)$$

4. Triangular Equilibrium Points

$$x = \mu - \frac{1}{2} + \frac{3p}{8} + \frac{1}{8\mu}(4-\mu)\sigma_{11} - \frac{1}{8\mu}(4+3\mu)\sigma_{21} \qquad (5)$$

$$-\tfrac{1}{8(1-\mu)}(3+\mu)\sigma_{12} + \tfrac{1}{8(1-\mu)}(7-3\mu)\sigma_{22} - \tfrac{1}{2}A_1$$

$$y = \pm\frac{\sqrt{3}}{2}[1 + \frac{2}{3}\left\{-\frac{p}{3} + \frac{1}{8\mu}(4 - 23\mu)\sigma_{11}\right\} \qquad (6)$$

$$+\tfrac{1}{8\mu}(-4 + 19\mu)\sigma_{21} + \tfrac{1}{8(1-\mu)}(-19 + 23\mu)\sigma_{12} + \left(\tfrac{1}{8(1-\mu)}(15 - 19\mu)\sigma_{22} - \tfrac{1}{2}A_1\right)]$$

$$D = \left\{+\frac{3}{16(1-\mu)}(12 - 43\mu + 23\mu^2)\sigma_{22} - \frac{3}{16}(2 - 8\mu)A_1\right\} \qquad (7)$$

Also, using the Jacobi constant, we have

$$C = 2\Omega = 2\bar{I}\xi^2 + 2\bar{m}\eta^2 + 2\bar{n} \qquad (8)$$

Hence, it follows that the above curve is an ellipse and the direction α of the major axis is given by equation (8). The length of the semi–major and semi–minor axes are given by

$$a_{sm} = \left(\frac{C - 2\bar{n}}{2\bar{I}}\right)^{\frac{1}{2}} and \quad a_{sm} = \left(\frac{C - 2\bar{n}}{2\bar{m}}\right)^{\frac{1}{2}} \qquad (9)$$

5. CONCLUSIONS

In this paper, we have studied the linear stability of triangular equilibrium points in the photogravitational restricted three body problem with triaxial rigid bodies, the bigger one an oblate spheroid and source of radiation.

(a) The co-ordinates of the triangular equilibrium points are given in equations (12) and (13). We see that the displacements of the new triangular equilibrium points from the classical triangular equilibrium points are small and depend upon the quantities

$\sigma_{1i} = B_{1i} - B_{3i}, \quad \sigma_{2i} = B_{2i} - B_{3i} (i = 1, 2)$
$\omega_1 = (1-p)(1-\mu), \quad \omega_2 = \mu,$
$B_{1i} = \frac{a_i^2}{5R^2}, B_{2i} = \frac{b_i^2}{5R^2}, B_{3i} = \frac{c_i^2}{5R^2} (i = 1, 2)$

where $a_i, b_i, c_i (i = 1, 2)$ the semi-axis of the triaxial rigid body and R is the distance between the primaries. (b) The mean motion 'n' of the primaries is given in the equation (4). (c) When both the bodies are spherical in shape (i = 1, 2), the results obtained are in agreement with those of the classical problem. (d) The stability of $L_{4,5}$ depends upon the value

$\mu_{crit} = 0.038520895 - -+0.81126474\sigma_{11} - 1.09626653\sigma_{21} - 0.02206859\sigma_{12} - 0.04071097\sigma_{22} - 0.1330773375A_1 - 0.0089174706p$

such that (i) For $0 \leq \mu < \mu_{crit}$ $L_{4,5}$ is stable. It may be noted that the range of stability decreases when compared to the classical case (ii) For $\mu_{crit} < \mu < 0.5$ $L_{4,5}$ is unstable and (iii) For $\mu = \mu_{crit}$ $L_{4,5}$ is unstable. (e) We also see that near the triangular equilibrium points there are long or short periodic elliptical orbits for the mass parameter $0 \leq \mu \leq \mu_{crit}$; the direction α of the major axis of the ellipse is given by $tan2\alpha = \frac{N}{D}$ where N and D are given by equation (7). We have also calculated the lengths of the semi-major and semi-minor axes of the ellipse given by equation (9).

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REFERENCES

Douskos, C. N. & Perdios, E. A., 2002, On the Stability of Equilibrium Points in the Relativistic Restricted Three-body Problem, CeMDA, 52, 312

Ishwar, B., 1997, Non-linear Stability in the Generalized Restricted Three Body Problem, CeMDA, 65, 253

Ishwar, B. & Elipe, A., 2001, Secular Solutions at Triangular Equilibrium Point in the Generalise Photogravitational Restricted Three–Body Problem, Ap&SS, 277, 437

Sanjay, J., Ajay, K., & Bhatnagar, K. B., 2009, Periodic Orbits Around the Collinear Libration Point in the Restricted Three-body Problem When the Smaller Primary is a Triaxial Rigid Body and Bigger Primary is a Source of Radiation Pressure, Indian J.phys., 83 (2), 171

Khanna, M. & Bhatnagar, K. B., 1998, Existence and Stability of Libration Points in the Restricted Three-body Problem when Thesmaller Primary is a Triaxial Rigid Body, Indian J. Pure. Appl. Math., 29 (10), 1011

Kalantonis,V.S., Perdios, E.A., & Perdiou,A.E., 2008, The Sitnikov Family and the Associated Families of 3D Periodic Orbits in the Photogravitational RTBP with Oblateness, ApSS, 315, 323

Avdhesh, K. & Ishwar. B., 2012, Linear Stability of Triangular Equilibrium Points in the Photogravitational Restricted Three Body Problem with Triaxial Rigid Bodies and Oblate Spheroid, Nova Science Publishers, 6, Number 1, (JNSST 2012)

Lara, M. & Elipe, A., 1997, Periodic Orbits in the Restricted Three Body Problem with Radiation Pressure, CeMDA, 68, 1

Mittal, A., Ahmad, I., & Bhatnagar, K. B., 2009, Periodic Orbits Generated by Lagrangian Solutions of the Restricted Three Body Problem When One of the Primaries is an Oblate Body, Ap&SS, 319, 63

Papadakis.K. E. & Kanavos.S. S., 2007, Numerical Exploration of the Photogravitational Restricted Five- body Problem, Ap&SS, 310, 119