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CONSTRAINTS ON PRE-INFLATION COSMOLOGY AND DARK FLOW

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ABSTRACT

If the present universe is slightly open then pre-inflation curvature would appear as a cosmic dark-flow component of the CMB dipole moment. We summarize current cosmological constraints on this cosmic dark flow and analyze the possible constraints on parameters characterizing the pre-inflating universe in an inflation model with a present-day very slightly open ΛCDM cosmology. We employ an analytic model to show that for a broad class of inflation-generating effective potentials, the simple requirement that the observed dipole moment represents the pre-inflation curvature as it enters the horizon allows one to set upper and lower limits on the magnitude and wavelength scale of pre-inflation fluctuations in the inflaton field and the curvature parameter of the pre-inflation universe, as a function of the fraction of the total initial energy density in the inflaton field. We estimate that if the current CMB dipole is a universal dark flow (or if it is near the upper limit set by the Planck Collaboration) then the present constraints on ΛCDM cosmological parameters imply rather small curvature $\Omega_k \sim 0.1$ for the pre-inflating universe for a broad range of the fraction of the total energy in the inflaton field at the onset of inflation. Such small pre-inflation curvature might be indicative of open-inflation models in which there are two epochs of inflation.

Key words: cosmology: early universe – Inflation – cosmology: observations – cosmology: theory – cosmic microwave background

1. INTRODUCTION

In this work (Mathews et al., 2014a) we consider that the present universe is slightly open, i.e. $\Omega_0 \geq 0.994$ at the 95% confidence level. In this case, any curvature that existed before inflation could now be visible on the horizon. Such pre-inflation fluctuations in the inflaton field could appear as a cosmic dark (or bulk) flow cosmological flow. We summarize an analytic model (Kurki-Suonio et al., 1991; Mathews et al., 2014a) for an open cosmology with an adiabatic planar inhomogeneity of wavelength less than the initial Hubble scale. We show that one can utilize the CMB dipole and current cosmological parameters to fix the amplitude and wavelength of fluctuations as a function of energy content of the inflaton field as the universe just entered the inflation epoch.

A detection of a bulk flow would be interesting as it could be a remnant of the birth of the universe out of the M-theory landscape (Mersini-Houghton & Holman, 2009), or a remnant of multiple field inflation (Turner, 1991; Langlois & Piran, 1996). Of particular interest to the present work, however, is the possibility that a large-scale dipole moment in the universe could be a remnant of pre-inflation fluctuations from any source, but just visible on the horizon now.

It has been known since the 1980s (Lynden-Bell et al., 1988) that the local dipole flow extends to well beyond the local super cluster. This was dubbed “the great attractor”. However, subsequent work (Mathewson, 1992) has established that the local flow extends at least to 130 h^{-1} Mpc. Moreover, there is no evidence of infall into the “great attractor” at larger distances. There is, however, recent evidence (Tully et al., 2014) of a supercluster extending to a scale ∼160 Mpc. Although there remains, a need observationally analyze bulk flow at distances beyond ∼160 Mpc^{-1}, it has been demonstrated (Mathews et al., 2014b) that it is impossible to detect dark flow beyond about 150 Mpc via the galactic distance redshift relation.

Attempts have been made (Kashlinsky et al., 2010, 2011, 2012) to observationally detect such dark flow by means of the kinetic Sunayev-Zeldovich (KSZ) effect. A detailed analysis of the KSZ effect based upon the WMAP data (Hinshaw et al., 2013) seemed to confirm that a dark flow exists out to at least 800 h^{-1} Mpc (Kashlinsky et al., 2012). However, the constraints set by the Planck Collaboration (Planck CollaborationXIII, 2013) are consistent with dark flow to a (95% confidence level) upper limit of 254 km s^{-1}.

2. MODEL

We consider fluctuations in a scalar inflaton field. For simplicity, we assume adiabatic fluctuations, i.e the density variations in all forms of matter and energy including the scalar field have equal fractional over/under densities. The energy density of a general inhomogeneous inflaton field is

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2a^2}\nabla^2\phi^2 + V(\phi) \quad . \tag{1}$$

We will assume that the $\dot{\phi}^2/2$ term can dominate over $V(\phi)$ initially, but eventually $V(\phi)$ will dominate as inflation commences. The quantity most affected initially by the density perturbation in the scalar field is, therefore, the kinetic $\dot{\phi}^2/2$ term as inflation begins.

We consider a broad range of general inflation-generating potentials $V(\phi)$ to drive inflation (Liddle & Lyth, 2000) with the only restriction that they be continuously differentiable in the inflaton field ϕ, i.e. $dV/d\phi \neq 0$. We also restrict ourselves to modest fluctuations in the scalar field with a wavelength less than the initial Hubble scale. This allows one to ignore the gravitational reaction to the inhomogeneities. This allows one to approximate the initial expansion with fluctuation perturbations on top the usual LFRW metric.

The particle horizon is given by the radial null geodesic in these coordinates,

$$r_H(t) = a(t)\int_0^t \frac{dt'}{a(t')} \quad . \tag{2}$$

This is to be distinguished from the Hubble scale H^{-1}, which at any epoch is given by the Friedmann equation to be:

$$\frac{1}{H(t)} = a(t)\sqrt{1-\Omega(t)} \quad . \tag{3}$$

For small adiabatic inhomogeneities, the coupled equations for the Friedmann equation and the inflation can then be written

$$H^2 = \frac{8\pi}{3m_{Pl}^2}(\rho_m + \langle\rho_\phi\rangle) + \frac{1}{a^2} \quad , \tag{4}$$

$$\ddot{\phi} = \frac{1}{a^2}\nabla^2\phi - 3H\dot{\phi} - V'(\phi) \ , \tag{5}$$

where $H = H(t) = \dot{a}/a$ is the Hubble parameter, and $\phi = \phi(t,x)$ is the inhomogeneous inflaton field in terms of comoving coordinate x. One can assume that the matter is relativistic in the pre-inflation post-Planck epoch, so that $\rho_m = \rho_{m,i}(a_i/a)^4$ with $\rho_{m,i}$ the initial mass-energy density in the matter field. The brackets $\langle\rho_\phi\rangle$ denote the average energy density in the inflaton field. That is, we decompose the energy density in the inflaton field into an average part and an adiabatic fluctuating part.

$$\rho_\phi = \langle\rho_\phi\rangle + \delta\rho_\phi \quad . \tag{6}$$

We presume that the initial adiabatic inhomogeneities are determined at or near the Planck time. Hence, we set the initial Hubble scale equal to the Planck length. Plane-wave inhomogeneities in the inflaton field are written

$$\phi(t,z) = \phi_i + \delta\phi_i \sin\frac{2\pi}{\lambda_i}(a_i z - t) \quad , \tag{7}$$

where the wavelength of the fluctuation can then be parameterized (Kurki-Suonio et al., 1991) by ,

$$\lambda_i = lH_i^{-1} = \frac{l}{m_{Pl}} = l\sqrt{1-\Omega_i}a_i \quad , \tag{8}$$

with l dimensionless in the interval $0 < l < 1$.

The energy density in the initial inflaton field, $\rho_{\phi,i}$ is constrained to be less than the Planck energy density. This implies,

$$\rho_{\phi,i} < f\Omega_i\frac{3m_{Pl}^4}{8\pi} \quad , \quad 0 < \Omega_i < 1 \quad , \quad 0 < f < 1 \quad , \tag{9}$$

where $1-\Omega_i$ is the initial curvature in the pre-inflation universe, and f is the fraction of the initial total energy density in the inflaton field. If the largest inhomogeneous contribution is from the $\dot{\phi}^2/2$ term, then the amplitude of the inhomogeneity in Eq. (7) is constrained to be,

$$\frac{\delta\phi_i}{m_{Pl}} < \left(\frac{3f\Omega_i l^2}{16\pi^3}\right)^{1/2} \quad . \tag{10}$$

The maximum initial amplitude we consider is therefore

$$(3/16\pi^3)^{1/2}m_{Pl} = 0.078m_{Pl} \quad , \tag{11}$$

for fluctuations initially of a Hubble length.

Fluctuations beyond the Hubble scale can of course have larger amplitudes, but those are not considered here. Note, that the assumption of ignoring the effect of gravitational perturbations on the inflaton field in Eq. (5) is justified as long as we restrict ourselves to fluctuations less than the Hubble scale $H\lambda < 1$.

At the initial time t_i, we have $H_i\lambda_i \equiv l < 1$. After that the comoving wavelength $H\lambda$ decreases until inflation begins. During inflation then $H\lambda$ increases until a time t_x at which $H_x\lambda_x = 1$. At this time, the fluctuation exits the horizon and is frozen in until it re-enters the horizon at the present time. How much $H\lambda$ decreases during the time interval from t_i to t_x depends upon the initial closure parameter Ω_i (Kurki-Suonio et al., 1991).

The problem, therefore, has three cosmological parameters, Ω_i, l, and f, plus parameters related to the inflaton potential $V(\phi)$. An analytic model has been developed (Mathews et al., 2014a) to show that the inflaton potential can be constrained from the COBE (Smoot et al., 2013) normalization of fluctuations in the CMB for any possible differential inflaton potential. We also show that the initial wavelength parameter l and the initial closure Ω_i can be constrained for a broad range of scalar-field energy-density contributions f by two requirements. One is that the resultant dipole anisotropy does not exceed the currently observed CMB dipole moment. The other is that the resulting dipole is in excess of the background due to isocurvature quantum fluctuations generated during inflation.

To begin with, the equation of state for the total density in Eq. (4) can be approximated as

$$\rho_m + \langle \rho_\phi \rangle \approx A\left(\frac{a_i}{a}\right)^4 + B \quad , \tag{12}$$

where $A = \rho_{m,i}$, and $B = (3m_{pl}/8\pi))V(\phi_i)$ are constants. Explicitly, from t_i to t_x, we invoke the slow-roll approximation. Another simplifying assumption is that $V(\phi) \sim B$ is initially small compared to the matter density for the first scale (the one we are interested) to cross the horizon. This assumption was verified in Kurki-Suonio et al. (1991) by a numerical of the equations of motion.

With these assumptions, the solution (Kurki-Suonio et al., 1991) of Eq. (4) for the scale factor at horizon crossing is simply,

$$\left(\frac{a_x}{a_i}\right) = \left(\frac{1 - l^2(1-\Omega_i)}{Bl^2}\right)^{1/2} \quad . \tag{13}$$

This analytic approximation was also verified to be accurate to a few percent by detailed numerical simulations in Kurki-Suonio et al. (1991). We are interested in length scales of these fluctuations that have the minimal amount of inflation such that the pre-inflation horizon is just visible on the horizon now.

Ignoring the gradient terms and using the slow-roll condition, the amplitude when a fluctuation exits the horizon can be written

$$\left.\frac{\delta\rho}{\rho + p}\right|_x \approx K\frac{(\sqrt{f\Omega_i}l^2}{[1 - l^2(1-\Omega_i)]^{3/2}} \quad , \tag{14}$$

where the constant K is given by:

$$K = \left[1 + \frac{3}{2\pi}\right] 8\pi\sqrt{2}\frac{V(\phi_i)^{3/2}}{V'(\phi_i)m_{Pl}^3} \ . \tag{15}$$

The normalization of the inflaton potential in Eq. (15) can be fixed from the quantum isocurvature fluctuations generated during inflation. Assuming that the only contribution to the observed CMB power spectrum is from adiabatic density perturbations, then the COBE (Smoot et al., 2013) normalization ($\delta_H = 1.91 \times 10^{-5}$) of the CMB power spectrum requires (Liddle & Lyth, 2000) ,

$$\frac{V^{3/2}(\phi_x)}{m_{Pl}^3 V'(\phi_x)} = 5.20 \times 10^{-4} \quad . \tag{16}$$

We then deduce the constant K, independently of the analytic form of the potential,

$$K = 5.2 \times 10^{-4}\left[1 + \frac{3}{2\pi}\right] 8\pi\sqrt{2} \ = \ 0.0270. \tag{17}$$

The constraint on pre-inflation fluctuation parameters from the requirement that the observed fluctuations exceed the magnitude of quantum fluctuations is then,

$$\frac{(\sqrt{f\Omega_i}l^2}{[1 - l^2(1-\Omega_i)]^{3/2}} > \delta_H/K \sim 7.1 \times 10^{-4} \quad . \tag{18}$$

2.1. *Constraint from Ω_0 in Ω_Λ, and Ω_m*

Eq. (3) directly relates the present Hubble scale to the present value of Ω_0,

$$(1 - \Omega_0)^{1/2} = \frac{1}{H_0} \quad , \tag{19}$$

However, the pre-inflation comoving Hubble scale should appear on the present horizon, r_h. In an open ΛCDM universe, the largest observable CMB scale has the comoving size

$$\frac{r_l}{a_0} = \frac{1}{H_0}\int_0^1 \frac{dx}{\sqrt{\Omega_\Lambda x^4 + (1-\Omega_0)x^2 + \Omega_m x + \Omega_\gamma}} \quad . \tag{20}$$

For a nearly flat $\Omega_0 = 0.994$ cosmology we can adopt values $\Omega_\Lambda = 0.697$, and $\Omega_m = 0.297$, (with $\Omega_\gamma = 0$) that are consistent with the Planck (Planck CollaborationXVI, 2013) and WMAP (Hinshaw et al., 2013) results. For these parameters, then $r_l/a_0 \approx 3.3/H_0$.

Equating the pre-inflation comoving Hubble scale with the current horizon we have the following relation between Ω_0 and parameters Ω_i and l

$$l\sqrt{(1-\Omega_i)} \approx 3.3\sqrt{(1-\Omega_0)} \quad . \tag{21}$$

2.2. *Constraint from CMB Dipole*

The observed CMB temperature is $T = 2.7258 \pm 0.00057$ (Fixen, 2009). The magnitude of the dipole moment corrected (Kogut, et al., 1993) to the frame of the Local Group, is 5.68 mK. Hence, we have

$$\left.\frac{\delta\rho}{\rho + p}\right|_{dipole} = 3\frac{\delta T}{T} \approx 6.25 \times 10^{-3} \quad . \tag{22}$$

Equating this with Eqs. (14) and (15), then this gives the constraint

$$\frac{\sqrt{(f\Omega_i)}l^2}{[1 - l^2(1-\Omega_i)]^{3/2}} < \frac{3}{K}\frac{\delta T}{T} \sim 0.23 \quad , \tag{23}$$

This is an upper limit since it would correspond to the case where all of the observed CMB dipole is due to a pre-inflation dark flow, with no contribution from the motion of the Local Group with respect to the background CMB. For example, the Planck Collaboration upper limit of < 254 km s^{-1} (Planck CollaborationXIII, 2013) for a bulk flow velocity would imply an upper limit of the pre-inflation dipole moment to be $\delta\rho/\rho + p < 2.5 \times 10^{-3}$ leading to an upper limit of $\sqrt{(f\Omega_i)}l^2/[1 - l^2(1-\Omega_i)]^{3/2} < 0.1$ in Eq. (23).

3. RESULTS

Combining Eqs. (18), (21), and (23), one can see that there are unique values for Ω_i and l for each value of f that satisfy these inequalities. These are summarized in Figure 1 from Mathews et al. (2014a). The upper solid lines show the upper limits to Ω_i based upon the CMB-dipole or the Planck limit to the dark-flow velocity. The lower solid line is the lower limit to Ω_i from the

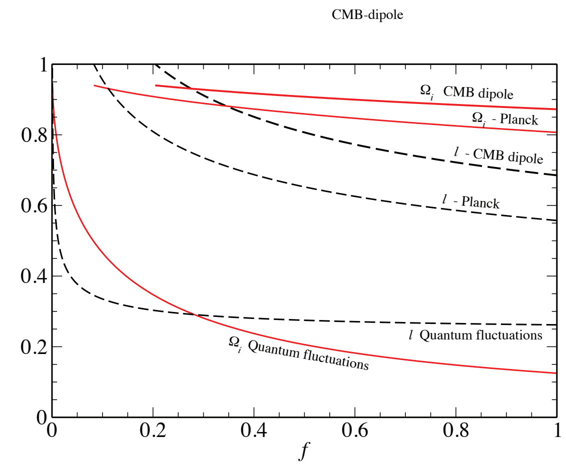

Figure 1. (color online) Constraints on the pre-inflation parameters as a function of the fraction f of the initial pre-inflation energy density in the inflaton field. Solid lines are for the initial closure parameter Ω_i. Dashed lines show the wavelength parameter l for pre-inflation fluctuations in the scalar field. Upper curves are maximum values based upon assuming that the entire observed CMB dipole with respect to the Local Group is due to a pre-inflation fluctuation, or assuming the Planck upper limit to the dark-flow velocity. Lower curves are minimum values from the requirement that such fluctuations be visible above the inflation-generated quantum fluctuations.

requirement that it exceed inflation generated quantum fluctuations. Similarly, the dashed lines show upper and lower limits to the wavelength parameter l as labeled. So for example, if the observed CMB dipole is mainly due to a cosmic dark flow (Kashlinsky et al., 2012), then a rather large fluctuation wavelength, $l \sim 0.7 - 1$ and small pre-inflation curvature $(1 - \Omega_i) \sim 0.1$ is implied for a broad range of the possible initial contribution f of the inflaton scalar field to the total pre-inflation energy density. Note, that $f < 0.2$ is not allowed in this case from the requirement that $l < 1$.

4. CONCLUSIONS

We have analyzed (Mathews et al., 2014a) a chaotic open inflationary universe with plane-wave adiabatic fluctuations in the scalar field characterized by a general inflaton potential. We have shown in a simple analytic model that such fluctuations are constrained by the requirement that they exceed the background quantum isocurvature fluctuations in the CMB. They are also constrained by the near flatness of the current universe, while an upper limit on the pre-inflation parameters is obtained in the limit that such fluctuations appear on the present horizon with an amplitude as a "dark flow" universal CMB dipole.

If the present CMB dipole were to be established as a dark flow, or if the dark flow velocity were near the Planck 95% upper limit, that would constrain the pre-inflation closure parameter to be $\simeq 0.9$ for a broad range $(f > 0.2)$ of the fraction of mass-energy in the inflaton field as the inflation epoch began. Such a small value for the pre-inflation curvature $(1 - \Omega_i) \simeq 0.1$ could be suggestive of many possible open inflation models (Liddle & Lyth, 2000) in which there are two inflationary epochs.

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REFERENCES

Atrio-Barandela, F., 2013, On the Statistical Significance of the Bulk Flow Measured by the Planck Satellite, A&A, 557A, 116A

Fixen, D. J., 2009, The Temperature of the Cosmic Microwave Background, ApJ, 707, 916

Grischuk, L. & Zel'dovich, Y. B. 1978, Long-wavelength Perturbations of a Friedmann Universe, and Anisotropy of the Microwave Background Radiation, SvA, 22, 125

Hinshaw, G., et al., (*WMAP Collaboration*), 2013, ApJ, 208, 19

Kashlinsky, A., Atrio-Barandela, F., Ebeling, H., Edge, A., & Kocevski, D., 2010, A New Measurement of the Bulk Flow of X-Ray Luminous Clusters of Galaxies, ApJ, 712, L81

Kashlinsky, A., Atrio-Barandela, F., & Ebeling, H., 2011, Measuring the Dark Flow with Public X-ray Cluster Data, ApJ, 732, 1

Kashlinsky, A., Atrio-Barandela, F., & Ebeling, H., 2012, Measuring Bulk Motion of X-ray Clusters Via the Kinematic Sunyaev-Zeldovich Effect: Summarizing the "Dark Flow" Evidence and its Implications, PhRv, arXiv, 1202, 0717

Kashlinsky, A., Tkachev, I. I., & Frieman, J., 1994, Microwave Background Anisotropy in low-0 Inflationary Models and the Scale of Homogeneity in the Universe, PhRvL, 73, 1582

Kogut, A., et al., 1993, Dipole Anisotropy in the COBE Differential Microwave Radiometers First-Year Sky Maps, ApJ, 419, 1

Kurki-Suonio, H., Graziani, F., & Mathews, G. J., 1991, Prospects for Observing Subhorizon Preinflation Fluctuations in the Cosmic Microwave Background, PhRvD, D44, 3072

Langlois, D. & Piran, T., 1996, Cosmic Microwave Background Dipole From an Entropy Gradient, PhRvD, 53(6), 2908

Liddle, A. R. & Lyth, D. H., 2000, *Cosmological Inflation and Large Scale Structure*, Cambridge University Press, Cambridge, UK

Lynden-Bell, D., et al., 1988, Spectroscopy and Photometry of Elliptical Galaxies. V - Galaxy Streaming Toward the New Supergalactic Center, ApJ, 326, 19

Mathews, G. J., Lan, N. Q., & Kajino. T., 2014, Constraints on Pre-inflation Fluctuations in a Nearly Flat Open ΛCDM Cosmology, arXiv, 1406, 3409M

Mathews, G. J., Rose, B., Garnavich, P., Yamazaki, D., & Kajino. T., 2014, arXiv, 1412, 1529M

Mathewson, D. S., Ford, V. L., & Buchhorn, M., 1992, A Southern Sky Survey of the Peculiar Velocities of 1355 Spiral Galaxies, ApJS, 81, 413
Mersini-Houghton, L. & Holman, R., 2009, "Tilting" the Universe with the Landscape Multiverse: the Dark Flow, JCAP, 2, 6
Planck Collaboration, & Planck III, 2013, Planck Intermediate Results. XIII. Constraints on Peculiar Velocities, A&A, arXiv 1303, 5090
Planck Collaboration, Planck VI, 2013, Planck Intermediate Results. XVI. Profile Likelihoods for Cosmological Parameters, A&A Submitted, arXiv, 1311, 1657v2
Smoot, G. F., et al., 1992, Structure in the COBE Differential Microwave Radiometer First-year Maps, ApJL, 396, L1
Tully, R. B., Courtois, H., Hoffman, Y., & Pomarède, D., 2014, The Laniakea Supercluster of Galaxies, Nature, 513, 71
Turner, M. S., 1991, Tilted Universe and OtherRremnants of the Preinflationary Universe, PhRvD, 44, 3737