FORMULATION AND CONSTRAINTS ON LATE DECAYING DARK MATTER

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ABSTRACT

We consider a late decaying dark matter model in which cold dark matter begins to decay into relativistic particles at a recent epoch (z ≤ 1). A complete set of Boltzmann equations for dark matter and other relevant particles is derived, which is necessary to calculate the evolution of the energy density and density perturbations. We show that the large entropy production and associated bulk viscosity from such decays leads to a recently accelerating cosmology consistent with observations. We determine the constraints on the decaying dark matter model with bulk viscosity by using a MCMC method combined with observational data of the CMB and type Ia supernovae.

Key words: cosmology: dark matter - cosmology: dark energy

1. INTRODUCTION

For more than a decade, modern cosmology has been faced with the dilemma that most of the mass-energy in the universe is attributed to material of which we know almost nothing. It has been a difficult to understand and explain the nature and origin of both the dark energy responsible for the present apparent acceleration and the cold dark matter responsible for most of the gravitational mass of galaxies and clusters. The simple coincidence that both of these unknown entities currently contribute comparable mass energy towards the closure of the universe begs the question of whether they could be different manifestations of the same physical phenomenon. Indeed, suggestions along this line have been made by many.

In previous work Mathews et al. (2008) it was proposed that a unity of dark matter and dark energy might be explained if the dark energy could be produced from the bulk viscosity induced by a delayed decaying dark-matter particle. That work demonstrated that if dark-matter particles begin to decay to relativistic particles near the present epoch, this would produce a cosmology consistent with the observed cosmic acceleration deduced from the type Ia supernova distance-redshift relation, without the need for a cosmological constant. Hence, this paradigm has the possibility to account for the apparent dark energy without the well known fine tuning and smallness problems associated with a cosmological constant. In addition, in this model the apparent acceleration is a temporary phenomenon. This avoids some of the difficulties in accommodating a cosmological constant in string theory. This model thus shifts the dilemma of modern cosmology from that of explaining dark energy to one of explaining how an otherwise stable heavy particle might begin to decay at a late epoch.

That previous work, however, was limited in that it only dealt with the supernova-redshift constraint and the difference between the current content of dark matter content and that in the past. The previous work did not consider the broader set of available cosmological constraints obtainable from simultaneous fits to the cosmic microwave background (CMB), large scale structure (LSS), baryon acoustic oscillations, limits to H₀, and the matter power spectrum, along with the SNIa redshift distance relation. Although our decaying dark matter scenario does not occur during the photon decoupling epoch and the early structure formation epoch, it does affect the CMB and LSS due to differences in the look-back time from the changing dark matter/dark energy content at photon decoupling relative to the present epoch. Hence, in this work we consider a simultaneous fit to the CMB, as a means to constrain this paradigm to unify dark matter and dark energy. We deduce constraints on the parameters characterizing the decaying dark matter cosmology by using the Markov Chain Monte Carlo method applied to the 9 year CMB data from WMAP9 Komatsu et al. (2011).

This paper is organized as follows: In section II, we derive the background dynamic equations for the evolution of a universe with decaying dark matter. In section III, we describe the method to fit the CMB data. In the last section, we summarize the fitting results and conclusions.
2. COSMOLOGICAL MODEL

2.1. Candidates for Late Decaying Dark Matter

There are already strong observational constraints on the density of photons from any decaying dark matter, such as their effect on the re-ionization epoch. To avoid these observational constraints, the decay products must not include photons or charged particles that would be easily detectable (Yuksel et al. 2007). Neutrinos or some other light weakly interacting particle are perhaps the most natural products from such a decay. Admittedly, it is a weak point of this paradigm that one must contrive both a decaying particle with the right decay products and lifetime, and also find a mechanism to delay the onset of decay. Nevertheless, in view of the many difficulties in accounting for the dark energy (Carroll et al. 1992), it is worthwhile pursuing any possible scenario until it is either confirmed or eliminated as a possibility. That is the motivation of this work. In particular, in this paper we scrutinize this cosmological model on the basis of all observational constraints, not just the supernova data as in earlier works Mathews et al. (2008).

Although this model is somewhat contrived, there are at least a few plausible candidates that come to mind. Possible candidates for late decaying dark matter have been discussed elsewhere Mathews et al. (2008) and need not be repeated in detail here. Nevertheless, for completeness, we provide a partial list of possible candidates. A good candidate Wilson et al. (2007) is that of a heavy sterile neutrino. For example, sterile neutrinos could decay into light $\nu_e$, $\nu_\mu$, $\nu_\tau$ “active” neutrinos (Abazajian et al. 2001). Various models have been proposed in which singlet "sterile" neutrinos $\nu_s$ mix in vacuum with active neutrinos ($\nu_e$, $\nu_\mu$, $\nu_\tau$). Such models provide both warm and cold dark matter candidates. Because of this mixing, sterile neutrinos are not truly “sterile” and can decay. In most of these models, however, the sterile neutrinos are produced in the very early universe through active neutrino scattering-induced decoherence and have a relatively low abundance. It is possible Wilson et al. (2007), however, that this production process could be augmented by medium enhancement stemming from a large lepton number. Here we speculate that a similar medium effect might also induce a late time enhancement of the decay rate.

There are also other ways by which such a heavy neutrino might be delayed from decaying until the present epoch. One is a cascade of intermediate decays prior to the final bulk-viscosity generating decay, which is possible but difficult to make consistent with observational constraints Wilson et al. (2007). Fitting the supernova magnitude vs. redshift relation requires one of two other possibilities. One is a late low-temperature cosmic phase transition whereby a new ground state causes a previously stable dark matter particle to become unstable. For example, a late decaying heavy neutrino could be obtained if the decay is caused by some horizontal interaction (e.g. as in the Majoron $\chi_{\mu0}$ et al. (1980) or familion Wilczek (1982) models). Another possibility is that a time varying effective mass for either the decaying particle or its decay products could occur whereby a new ground state appears due to a level crossing at a late epoch. In the present context the self interaction of the neutrino could produce a time-dependent heavy neutrino mass such that the lifetime for decay of an initially unstable long-lived neutrino becomes significantly shorter at late times.

Another possibility might be a more generic long-lived dark-matter particle $\psi$ whose rest mass increases with time. This occurs, for example, in scalar-tensor theories of gravity by having the rest mass relate to the expectation value of a scalar field $\phi$. If the potential for $\phi$ depends upon the number density of $\psi$ particles, then the mass of the particles could change with the cosmic expansion leading to late-time decay.

Finally, supersymmetric dark matter initially produced as a superWIMP has been studied as a means to obtain the correct relic density. In this scenario, the superWIMP then decays to a lighter stable dark-matter particle. In our context, a decaying superWIMP with time-dependent couplings might lead to late-time decay. Another possibility is that the light supersymmetric particle itself might be unstable with a variable decay lifetime. For example Hamaguchi et al. (1998), there are discrete gauge symmetries (e.g. $\mathbb{Z}_{10}$) which naturally protect heavy $X$ gauge particles from decaying into ordinary light particles. Thus, the X particles are a candidate for long-lived dark matter. The lifetime of the $X$, however strongly depends on the ratio of the cutoff scale ($M_s \approx 10^{19}$ GeV) to the mass of the $X$.

$$\tau_X \sim \left( \frac{M_s}{M_X} \right)^{14} \frac{1}{M_X} = 10^2 - 10^{17} \, \text{Gyr} \ .$$

Hence, even a small variation in either $M_s$ or $M_s$ could lead to a change in the decay lifetime at late time.

2.2. Cosmic Evolution

The time evolution of an homogeneous and isotropic expanding universe with late decaying dark matter and bulk viscosity can be written as a modified Friedmann equation in which we allow for non-flat $k \neq 0$ and the usual cosmological constant $\Lambda$.

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3} \rho + \frac{\Lambda}{3} - \frac{k}{a^2}$$

where, $\rho$ is now composed of several terms

$$\rho = \rho_{DM} + \rho_b + \rho_r + \rho_{\chi} + \rho_{\nu} + \rho_{BV}$$

Here, $\rho_{DM}, \rho_b,$ and $\rho_r$ are the usual densities of stable dark matter, baryons, stable relativistic particles, and the standard cosmological constant vacuum energy density, respectively. In addition, we have added $\rho_{\chi}$ to denote the energy density of heavy decaying dark matter particles, $\rho_{\nu}$ to denote the energy density of light relativistic particles specifically produced by decaying dark matter, and $\rho_{BV}$ as the contribution from the bulk viscosity. These quantities $\rho_b$ and $\rho_r$ and $\rho_{BV}$ are given
by a solution to the continuity equation Mathews et al. (2008)

$$\rho_h = \rho_h(t_d) a^{-3} e^{-(t-t_d)/\tau_d} ,$$  \hspace{1cm} (4)

$$\rho_r = a^{-4} \lambda h \rho_h(t_d) \int_{t_d}^{t} e^{-(t'-t_d)/\tau_d} a(t') dt' ,$$  \hspace{1cm} (5)

$$\rho_{BV} = a^{-4} g \int_{t_d}^{t} H^2 a(t') \zeta(t') dt' ,$$  \hspace{1cm} (6)

where we have denoted $t_d$ as the time at which decay begins with a decay lifetime of $\tau_d$, and have set $\rho_r(t_d) = 0$ prior to the onset of decay.

Next, we write the Boltzmann equations for the distribution function of late decaying dark matter (LDDM) and light relativistic particles (LR). The Boltzmann equation for the distribution function of the LDDM $f_h(q_h(t))$ is

$$Df_h \over Dt = \partial f_h / \partial t + \partial f_h / \partial x^i \partial x^i + \partial f_h / \partial q_h \partial q_h + \partial f_h / \partial n_i \partial n_i = (df_h / dt)_c ,$$  \hspace{1cm} (7)

where $n_i$ is the unit vector pointed in the direction of the momentum.

Similarly, the Boltzmann equation of LR particles is

$$Df_r \over Dt = \partial f_r / \partial t + \partial f_r / \partial x^i \partial x^i + \partial f_r / \partial q_r \partial q_r + \partial f_r / \partial n_i \partial n_i = (df_r / dt)_c .$$  \hspace{1cm} (8)

Now, in addition to the usual contributions to the closure density from the cosmological constant $\Omega_{\Lambda} = \Lambda/3H_0^2$, the relativistic particles and stable dark matter present initially

$$\Omega_{\gamma} = \frac{8\pi G \rho_{n0}/3H_0^2}{(1+z)^3} , \Omega_{DM} = \frac{8\pi G \rho_{DM}/3H_0^2}{(1+z)^3} ,$$  \hspace{1cm} (9)

and baryons, $\Omega_b = (8\pi G \rho_{n0}/3H_0^2)(1+z)^3$, one has contributions from the energy density in decaying cold dark matter particles $\Omega_h(z)$, relativistic particles generated from decaying dark matter $\Omega_r(z)$, and the cosmic bulk viscosity $\Omega_{BV}(z)$. Note that $\Omega_h$, $\Omega_r$ and $\Omega_{BV}$ all have a non-trivial dependence on redshift corresponding to equations (4) - (6).

3. STATISTICAL ANALYSIS WITH THE OBSERVATION DATA

Based upon the above description, there are three new cosmological parameters associated with this paradigm. These are the delay time $t_D$ at which decay begins, the decay lifetime, $\tau_D$, and the correction for nonlinear radiation transport $C$. These we now wish to constrain from observational data along with the rest of the standard cosmological variables. To do this we make use of the standard Bayesian Monte Carlo Markov Chain (MCMC) method as described in Lewis & Bridle (2002).

We have modified the publicly available CosmoMC package Lewis & Bridle (2002) to satisfy this decaying dark matter model as described above. Following the usual prescription we then determine the best-fit values using the maximum likelihood method. We take the total likelihood function $\chi^2 = -2\log L$ as the product of the separate likelihood functions of each data set and thus we write,

$$\chi^2 = \chi^2_{SN} + \chi^2_{CMB} .$$  \hspace{1cm} (10)

Then, one obtains the best fit values of all parameters by minimizing $\chi^2$.

3.1. Type Ia Supernova Data and Constraint:

We wish to consider the most general cosmology with both finite $\Lambda$, normal dark matter, and decaying dark matter. In this case the dependence of the luminosity distance to cosmological redshift is given by a slightly more complicated relation from the standard $\Lambda$CDM cosmology, i.e. we now have,

$$D_L = \frac{c(1+z)}{H_0} \left\{ \int_0^z [\Omega_{\Lambda} + \Omega_h(z') + \Omega_{DM}(z')]^{1/2} + \Omega_h(z') + \Omega_r(z') + \Omega_{BV}(z')]^{-1/2} \right\} ,$$  \hspace{1cm} (11)

where $H_0$ is the present value of the Hubble constant. This luminosity distance is related to the apparent magnitude of supernovae by the usual relation,

$$\delta m(z) = m(z) - M = 5\log_{10}[D_L(z)/\text{Mpc}] + 25 ,$$  \hspace{1cm} (12)

where $\delta m(z)$ is the distance modulus and $M$ is the absolute magnitude, which is assumed to be constant for type Ia supernovae standard candles. The $\chi^2$ for type Ia supernovae is given by Amanullah et al. (2010)

$$\chi^2_{SN} = \sum_{i,j=1}^{N} |\delta m(z_i)_{obs} - \delta m(z_i)^{th}| \times (C_{SN}^{-1})_{ij} |\delta m(z_i)_{obs} - \delta m(z_i)^{th}| .$$  \hspace{1cm} (13)

Here $C_{SN}$ is the covariance matrix with systematic errors.

3.2. CMB Constraint:

The characteristic angular scale $\theta_A$ of the peaks of the angular power spectrum in the CMB anisotropies is defined as Page et al (2003)

$$\theta_A = \frac{r(z_s)}{r(z_A)} = \frac{\pi}{l_A} ,$$  \hspace{1cm} (14)

where $l_A$ is the acoustic scale, $z_s$ is the redshift at decoupling, and $r(z)$ is the comoving distance at decoupling

$$r(z) = \frac{c}{H_0} \int_0^z \frac{dz'}{H(z')} .$$  \hspace{1cm} (15)

In the present model the Hubble parameter $H(z)$ is given by Eq. (2). The quantity $r_s(z_s)$ in Eq. (14) is the comoving sound horizon distance at decoupling. This is defined by

$$r_s(z_s) = \int_0^{z_s} \frac{(1+z)^2 R(z)}{H(z)} dz ,$$  \hspace{1cm} (16)
Table 1

<table>
<thead>
<tr>
<th>case</th>
<th>$l_A$</th>
<th>R</th>
<th>$z_*$</th>
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<tr>
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<td>29.698</td>
<td>-1.333</td>
</tr>
<tr>
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</tr>
<tr>
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<td>-113.18</td>
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Table 2

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<th>parameter</th>
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<th>$t_d$</th>
<th>$\Omega_b$</th>
<th>$\Omega_m$</th>
<th>$n_s$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>0.112 ± 0.01</td>
<td>10.5 ± 2</td>
<td>0.0225 ± 0.002</td>
<td>0.235 ± 0.001</td>
<td>0.0968 ± 0.0001</td>
<td>0.71 ± 0.01</td>
</tr>
</tbody>
</table>

where the sound speed distance $R(z)$ is given by Mangano et al. (2002)

$$R(z) = [1 + \frac{3 \Omega_0 h^2}{4 \Omega_0 h} (1 + z)]^{-1/2} \ ,$$  

(17)

where $\Omega_0 = 1 - \Omega_k$ is the total closure parameter.

For our purposes we can use the fitting function to find the redshift at decoupling $z_*$ proposed by Hu and Sugiyama Hu & Sugiyama (1996)

$$z_* = 1048[1 + 0.00124(\Omega_0 h^2) - 0.738][1 + g_1(\Omega_0 h^2)^{g_2}] \ ,$$  

(18)

where

$$g_1 = \frac{0.0783(\Omega_0 h^2)^{-0.238}}{1 + 39.5(\Omega_0 h^2)^{0.763}} \ \text{and} \ g_2 = \frac{0.56}{1 + 21.1(\Omega_0 h^2)^{1.81}} \ .$$  

(19)

The $\chi^2$ of the cosmic microwave background fit is constructed as $\chi^2_{CMB} = -2 l n L = \sum X^2(C^{-1})_{ij} X$ Komatsu et al. (2011), where

$$X^2 = (l_A - l_{A_{WAP}}, R - R_{A_{WAP}}, z_* - z_{A_{WAP}}) \ ,$$  

(20)

with $l_{A_{WAP}} = 302.09 \ , \ R_{A_{WAP}} = 1.725 \ , \ \text{and} \ z_{A_{WAP}} = 1091.3. \ \text{Table 1 shows the the inverse covariance matrix used in our analysis.}$

4. RESULTS AND CONCLUSIONS

We performed a MCMC analysis of a cosmological model with a bulk viscosity from decaying dark matter in the parameter space of $\Omega_b h^2, \Omega_m h^2, \Omega_D, h, \Omega_D h^2, \tau_*, \omega, n_s, m_1, t_{dd}, C$) All other parameters were fixed at values from the WMAP9 analysis. Table 2 summarizes the deduced cosmological parameters from this work.

In summary, we have studied the evolution of the delayed decaying dark matter model with bulk viscosity by using a MCMC analysis to fit the SNIa and CMB data. We have shown that comparable fits to that of the $\Lambda$CDM cosmology can be obtained, but at the price of introducing a background of hidden relativistic particles.

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