

## APPLICATIONS OF THE HILBERT-HUANG TRANSFORM ON THE NON-STATIONARY ASTRONOMICAL TIME SERIES

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### ABSTRACT

The development of time-frequency analysis techniques allow astronomers to successfully deal with the non-stationary time series that originate from unstable physical mechanisms. We applied a recently developed time-frequency analysis method, the Hilbert-Huang transform (HHT), to two non-stationary phenomena: the superorbital modulation in the high-mass X-ray binary SMC X-1 and the quasi-periodic oscillation (QPO) of the AGN RE J1034+396. From the analysis of SMC X-1, we obtained a Hilbert spectrum that shows more detailed information in both the time and frequency domains. Then, a phase-resolved analysis of both the spectra and the orbital profiles was presented. From the spectral analysis, we noticed that the iron line production is dominated by different regions of this binary system in different superorbital phases. Furthermore, a pre-eclipse dip lying at orbital phase  $\sim 0.6 - 0.85$  was discovered during the superorbital transition state. We further applied the HHT to analyze the QPO of RE J1034+396. From the Hilbert spectrum and the O - C analysis results, we suggest that it is better to divide the evolution of the QPO into three epochs according to their different periodicities. The correlations between the QPO periods and corresponding fluxes were also different in these three epochs. The change in periodicity and the relationships could be interpreted as the change in oscillation mode based on the diskoseismology model.

*Key words:* accretion, accretion disks – galaxies: Seyfert – X-rays: galaxies, binaries, individuals (SMC X-1, RE J1034+396)

### 1. NON-STATIONARY PHENOMENA

Compact stellar objects are usually found in binary systems. They often present non-stationary signals, i.e., quasi-periodic, aperiodic, or intermittent signals, owing to the presence of accretion disks. The time-scales of these variabilities spread over a wide range, from milliseconds to hundreds of days. An active galactic nucleus (AGN), a supermassive black hole lying in the center of an active galaxy, has many similar properties to stellar black holes. Moreover, all the emissions from AGNs are thought to be non-stationary due to the lack of a companion star.

### 2. HILBERT-HUANG TRANSFORM

The Hilbert-Huang transform (HHT, Huang et al., 1998) is optimized for analyzing non-stationary time series. Given a time series  $x(t)$ , the instantaneous frequency can be obtained using the Hilbert transform. However, it is meaningful only if the time series is an intrinsic mode function (IMF). We can yield a series of IMFs through ensemble empirical mode decomposition

(EEMD, Wu et al., 2009). The completeness of EEMD guarantees that

$$x(t) = \sum_{j=1}^n c_j(t) + r_n(t) \quad (1)$$

where  $c_j(t)$  are IMFs and  $r_n(t)$  is a residual. After applying the Hilbert transform on the IMFs, the original time series can be expressed as

$$x(t) = \sum_{j=1}^n a_j(t) \exp\left(i \int_0^t \omega_j(t') dt'\right) \quad (2)$$

where  $a_j(t)$  and  $\omega_j(t)$  are the instantaneous amplitudes and frequencies of IMFs.

### 3. SUPERORBITAL MODULATION IN SMC X-1

The intriguing superorbital modulation of SMC X-1 is thought to originate from an obscuration by a warped and tilted accretion disk with a period changing between 40 and 65 days (Clarkson et al., 2003). The quasi-periodicity is thought to be caused by the precession of accretion disk.

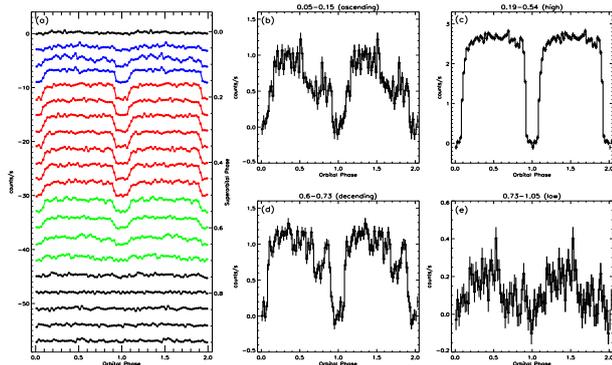


Figure 1. (a) Folded light curves of all the superorbital phases. (b) - (e) Folded light curves of different superorbital states. Adopted from Hu et al. (2013)

### 3.1. Time-Frequency Analysis

We applied the normalized Hilbert transform on the EEMD decomposed IMFs from the light curve collected by ASM/RXTE to yield a Hilbert spectrum. Comparing with the dynamic power spectrum, the Hilbert spectrum shows better resolution in both time and frequency domains. From the instantaneous frequency, we obtained a  $\sim 3200$  days recurrent time scale of the variability of superorbital modulation. Finally, using the instantaneous amplitude, we observed a positive correlation between superorbital period and amplitude.

### 3.2. Superorbital Phase-resolved Analysis

We further analysed the pointed observations made by PCA onboard RXTE. All the data were fitted by a soft black body component and a hard thermal Comptonized tail. We found no correlation between the plasma optical depth and the iron line equivalent width at the high intensity state but detected a clear positive correlation at the low intensity state. This indicates that the iron line production is dominated by different regions in different superorbital phases. The orbital profile was also found to vary with respect to superorbital phase. Figure 1 shows the superorbital phase-resolved orbital profiles. During the superorbital transition states, we observed a pre-eclipse dip indicating the existence of a bulge on outer accretion disk lying in orbital phase  $\sim 0.6 - 0.85$ .

## 4. THE QPO IN RE J1034+396

Quasi-Periodic Oscillation (QPO), commonly observed in black hole X-ray binaries (BHB), is a typical non-stationary phenomenon closely related to the Keplerian frequency of the innermost accretion disk. Active galactic Nuclei (AGN), as an upscaling of BHBs, are expected to exhibit QPOs. However, no significant QPO was detected until the first detection in the narrow-line Seyfert 1 AGN RE J1034+396 (Gierlinski et al., 2008).

We studied the QPO in RE J1034+396 using the data collected by XMM-Newton in 2007. Comparing with other time-frequency analysis methods, the Hilbert spectrum reveals the variation of the QPO period in great detail. Based on the HHT and O - C analysis,

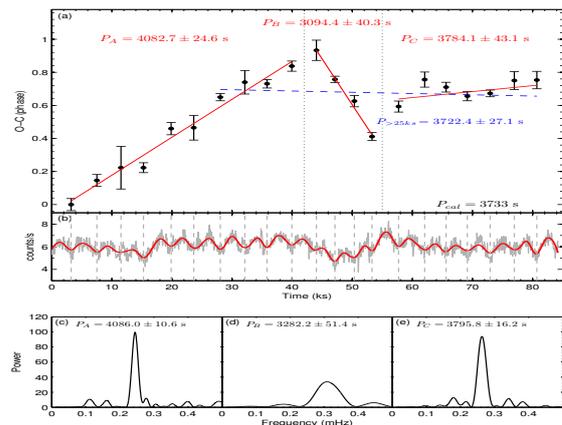


Figure 2. O - C result and the Lomb-Scargle power spectra for three epochs. Adopted from Hu et al. (2014)

we found that the evolution of the QPO can be divided into three epochs that show different characteristic periods (see Figure 2). The power spectra for individual epochs also agree with the O - C analysis result. We further found that the relation between the QPO period and X-ray flux before the presence of a flux-drop event significantly differs from that after this event. This discovery may indicate a change in oscillation mode based on the diskoseismology model (Wagoner et al., 2001).

## 5. SUMMARY & FUTURE WORK

With the help of the HHT, we successfully describe the variability of superorbital modulation in SMC X-1 and QPO in RE J1034+396 in great detail. To solve the missing link between AGNs and BHBs, further studies of the QPO behaviors, either low-frequency QPOs or high-frequency QPOs, are essential. Furthermore, we would also like to continue our focus on superorbital modulations of other X-ray binaries, like LMC X-4 and Her X-1, to deeply study the radiation-induced warp of accretion disk.

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